



Reliability Analysis of Reinforced Slopes Using Monte Carlo Simulation

H. Rahmati*, Seyed Morteza Marandi^b

^aPhD Student of Civil Engineering Department, Shahid Bahonar University of Kerman, Kerman, Iran
H.rahmati@eng.uk.ac.ir

^bProfessor of Civil Engineering Department, Shahid Bahonar University of Kerman, Kerman, Iran
marandi@uk.ac.ir

*Corresponding author

Abstract

Reliability analysis has gained special importance in geotechnical engineering in the past decades. One of the most important fields of geotechnical issues is the slopes stability. Slope stability analysis and their reliability analysis against slip and loading are important. Geotechnical engineers are also interested in improving and reinforcing weak slopes and increasing their factor of safety and reliability index. In this study, the uncertainties in soil strength parameters such as cohesion, internal friction angle and unit weight and force created in geogrid layers due to pull-out and bonding between soil and geogrid are considered. Then using Monte Carlo simulation Depending on the height of the slope, the reliability index is obtained in both of unreinforced and reinforced states. The critical slip surface of slope failure is also determined using PSO. The results of this analysis indicate the important role of reinforcements in increasing the reliability of soil slopes..

Keywords: Monte Carlo Simulation, Slope stability, reinforced slope, reliability analysis

INTRODUCTION 1.

In most structures and engineering designs, the pre-implementation studies and obtaining a comprehensive view of the possible hazards and considerations during implementation and operation is essential. In geotechnical engineering, these studies are more important because they deal with an unknown and high-risk area in soil and rock. Although the slope stability analysis using a single value for each parameter (mean of each soils parameter) determines a factor of safety for the slope studied, the heterogeneous nature of the soil and the dispersion of the parameter values even in close proximity make this possible that stochastic analysis differs from reality. The assumptions assumed in the analytical model in slope analysis, human error, etc., do not give us a complete view of the slope's stability coefficient [1-3].

Accordingly, compared to a deterministic analysis, probabilistic analysis with variable capability and uncertainty in input parameters would be a more useful way of examining slope stability issues and predicting soil behavior[4,5]. The concepts and technologies of soil reinforcement originate in prehistoric times. Straw, wood, and branches traditionally have been used to improve the quality of adobe bricks, to reinforce flower pots, and even to strengthen soil to control erosion. However, modern techniques for mechanically stabilizing soil were introduced in the 1960s. First used in France, a method known as "reinforced earth" used embedded narrow metal straps to reinforce soil[6]. In the United States this technique was adopted In 1972 by the California Division of Highways for construction of retaining walls. Many other methods on soil reinforcement were researched and implemented following the first applications in the United States[7]. The report examined the use of geogrids for the foundation of the passages and reported that the pressure clearing distribution and pressure distribution were directly related to the thickness and configuration of the geogrid reinforced foundation[8]. A typical solution to improve the bearing capacity of the soft clay slope is to remove part of the existing weak soil and replace it with granular soil (partial replacement). Depth of replacement fill depends on the required bearing capacity and permitted clearing. Sometimes this method results in high altitudes for soil replacement and therefore high costs. As a cost-effective alternative to this solution, geosynthetic reinforcement in the sand fill layer is provided. However, several studies have reported the behavior of foundations built in the sand-stabilized

slope [9-11]. Geosynthetics are modern materials used to improve soil conditions by providing tensile strength and stability. In the case of soil reinforcement, one of the main applications of geosynthetics is to enhance slope gradients. [12]. In addition, geosynthetic reinforcement is a cost-effective solution for fixing recurrent slope failures and constructing new permanent embankments [13,14].

In this study, in order to analyze a soil slope, in addition to using the deterministic method, Monte Carlo simulation method for reliability analysis was used. For this purpose, after assuming the hypothetical dimensions of the slope, the critical slope failure surface will be calculated using the pso method. In the next step, the slope is reinforced with geogrid sheets and the previous process is applied to obtain the reliability index in the reinforced state and the comparisons and results of these two modes are presented.

LIMIT EQUILIBRIUM METHOD 2.

There are various methods for analyzing slopes. Equilibrium method is the most popular method for analyzing slope stability. It should be noted that equilibrium methods, although widely extreme applicable, but one of the disadvantages of these methods, are not the consideration of the stress-strain relation in the soil. . According to Fig.1 for the sake of simplicity of calculation, interruptible forces are neglected and only the forces of weight and friction are investigated and the potential surface of failure, is assumed to be a circular arc with radius r and center O .

In order to determine the factor of safety and stability analysis in this method, the equilibrium forces and moment are considered on the desired part and the ratio of the resistance to the driving is obtained. According to Fig. 1, if the slip surface is considered part of a circle, it is possible to calculate the moment of resistance force and driven moments around the center of the circle.

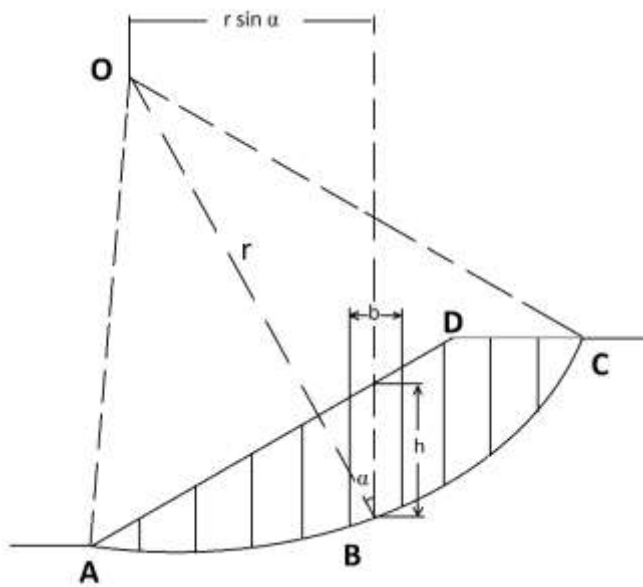


Fig.1. Segmentation of slope

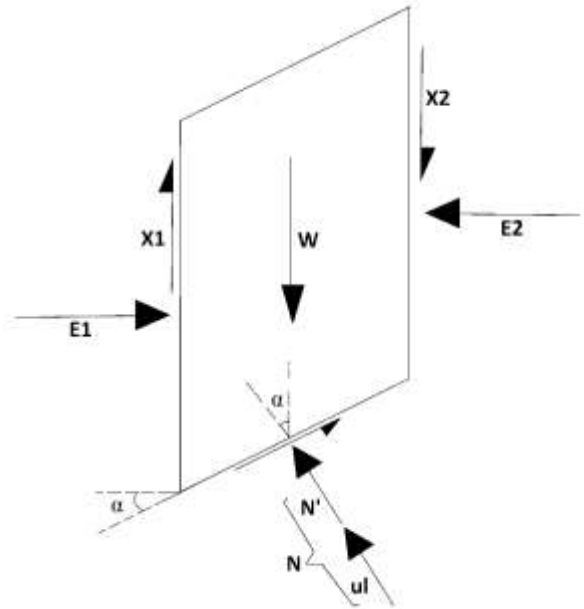


Fig.2. Sample slice

After segmentation, according to Fig.2 the forces applied to each segment must be determined and calculated and the sum of the resistive moments of all components is calculated. Similarly, driven moments are also calculated. And then, according to equation 1, the slope factor of safety is obtained.

$$FS = \frac{M_R}{M_D} \quad (1)$$

Were:

M_R : Is the existing resisting moment

M_D : Is the deriving moment

Also, any external force exerted on the slope must also be included in the calculations. The external force is classified as resistive or driven based on its role in stimulating the slope to break or resist failure. Accordingly, according to Equation 2, in a reinforced slope, the moments created by the armature members are added to the resistive moments and increase the safety factor.

$$FS_r = \frac{M_R + M_G}{M_D} = \frac{M_R + (T_{hor} \times Y)}{M_D} \quad (2)$$

Were:

M_G : is the reinforcement moment

T_{hor} : defined in Fig.3

Y : defined in Fig.3

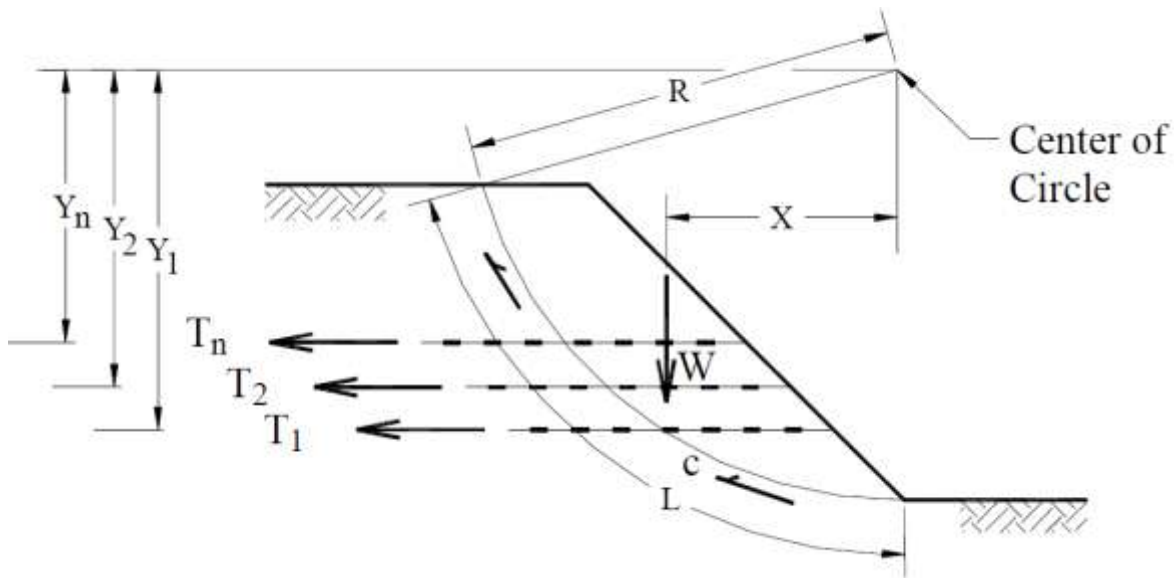


Fig.3. reinforcement layer in slope

As shown in Fig.3, the momentum that the reinforcement generates up on the slope is calculated around the center of the slip surface and contributes to the slope stability. In equation 2, three main parameters in slope stability analysis are presented, including M_R , M_G and M_D , which calculate these parameters as follows:

$$\sum M_D = \sum W r \sin \alpha \quad (3)$$

were:

W = the weight of soil mass, $W = \gamma b h$
 r = radius of failure surface

$$M_R = \sum \tau_f l = \sum (c' + \sigma' \tan \phi') l \quad (4)$$

were:

τ_f = shear stress on the surface
 l = the length of slip surface

$$M_G = \sum_1^n T_i \times Y_i \quad (5)$$

were T_i and Y_i is shown in Fig.3



CRITICAL SLIP SURFACE 3.

The Particle Swarm Optimization (PSO) method was used to obtain the critical failure surface. For this purpose, 2000 probable slip surface is considered, and the most critical arc is considered as the critical slip surface. In this paper the slip surface is considered as part of a circle and two parameters of slip center and the location of the fracture surface exit from the back of slope are considered and optimized as posterior parameters [16].

3. MONTE CARLO SIMULATION OF SLOPE

The use of probabilistic analysis in geotechnical engineering has increased in recent years. Many geotechnical issues due to dealing with the heterogeneous soil structure and the many uncertainties in computational parameters have to be analyzed by probabilistic analysis methods. In this regard, probabilistic analysis of soil slopes has been one of the most important geotechnical issues considered by engineers. The most important probabilistic methods used in geotechnical analysis, especially slope stability analysis, are: FORM, FOSM, PEM, JDRV and MCS [17-21]. The most prominent method is Monte Carlo reliability analysis. In this way, given the range of data for each input parameter, a lot of calculations are performed for all possible states with different values of the parameters. After performing Monte Carlo analysis and outputting all possible modes, output data analysis is performed and reliability index is calculated. The Monte Carlo simulation is a comprehensive and accepted method that even uses its results to verify other methods [22].

STOCHASTIC PARAMETER 4.

In the stability analysis uncertainty of slopes research, 4 input considered as random probabilistic parameters angle of soil (ϕ), weight (γ) and force (T_i). σ is standard that shows the parameters around the probability distribution above-mentioned distribution functions

$$F_{\phi}(\phi) = \frac{1}{\sigma_{\phi}\sqrt{2\pi}} \exp\left(-0.5\left(\frac{\phi - \phi_{mean}}{\sigma_{\phi}}\right)^2\right)$$

$$\phi_{min} \leq \phi \leq \phi_{max}$$

$$F_C(C) = \frac{1}{\sigma_C\sqrt{2\pi}} \exp\left(-0.5\left(\frac{C - C_{mean}}{\sigma_C}\right)^2\right)$$

$$C_{min} \leq C \leq C_{max}$$

$$F_{\gamma}(\gamma) = \frac{1}{\sigma_{\gamma}\sqrt{2\pi}} \exp\left(-0.5\left(\frac{\gamma - \gamma_{mean}}{\sigma_{\gamma}}\right)^2\right)$$

$$\gamma_{min} \leq \gamma \leq \gamma_{max}$$

$$F_{T_i}(T_i) = \frac{1}{\sigma_{T_i}\sqrt{2\pi}} \exp\left(-0.5\left(\frac{T_i - T_{i,mean}}{\sigma_{T_i}}\right)^2\right)$$

$$T_{i,min} \leq T_i \leq T_{i,max}$$

- (6) to answer the parameter in this parameters are variables. These are the internal friction cohesion (C), unit reinforcement tensile deviation of parameters dispersion of the mean. These random using ordinary functions (PDF). The random parameter are as follows:

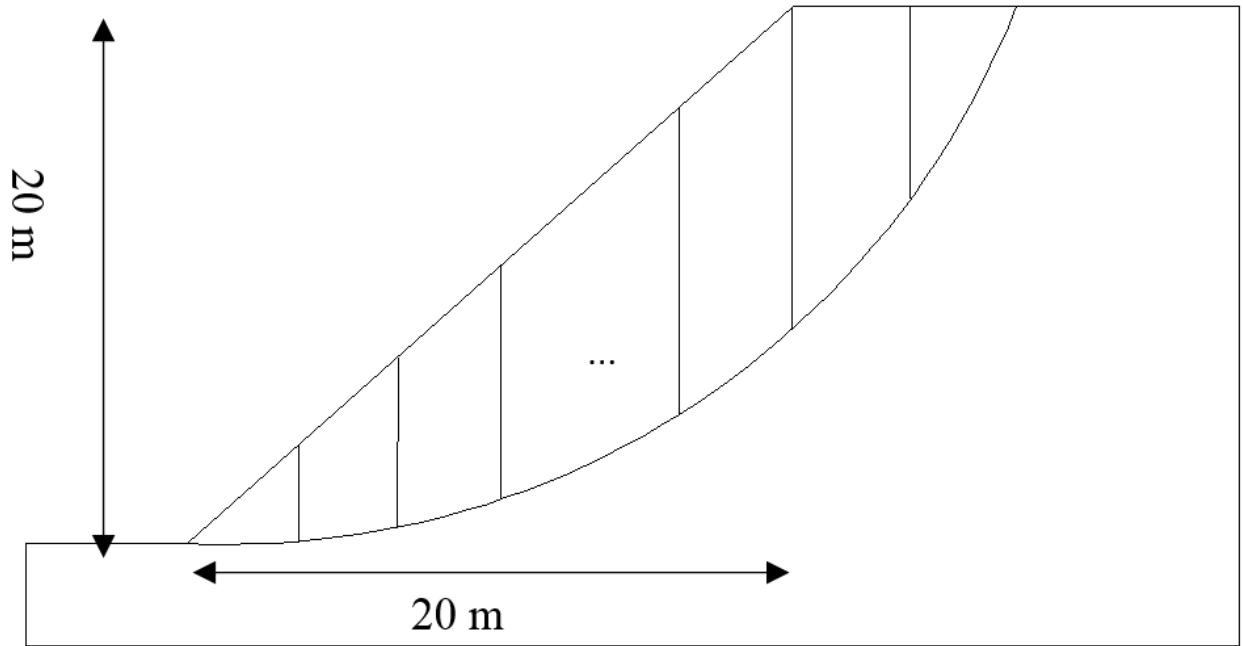


Fig.4. geometry of the selected slope

EXAMPLE SLOPE 5.

In this paper the reliability analysis is performed on an arbitrary slope. Figure 1 shows the geometrical characteristics of this slope. The length of slope is considered 20.0 m and slope's height is assumed to vary from 10.0 to 20.0 m, the maximum height 20.0 m as the final elevation discussed in this paper.

Based on a deterministic analysis that uses mean values of input parameter to determine the factor of safety. Using the PSO method and deterministic analysis, the critical slip surface as illustrated in Fig. 5 with blue line, obtained out of the 2000 possible failure mode.

Table 1. Arbitrary stochastic parameters with truncated normal distribution.

	Mean, μ	Standard deviation, σ
Unit weight (kN/m ³)	18.0	1.0
Cohesion (kPa)	15.0	1.5
Friction angle (Degree)	25.0	2.5
T_i (Kn/m)	20.0	1.0

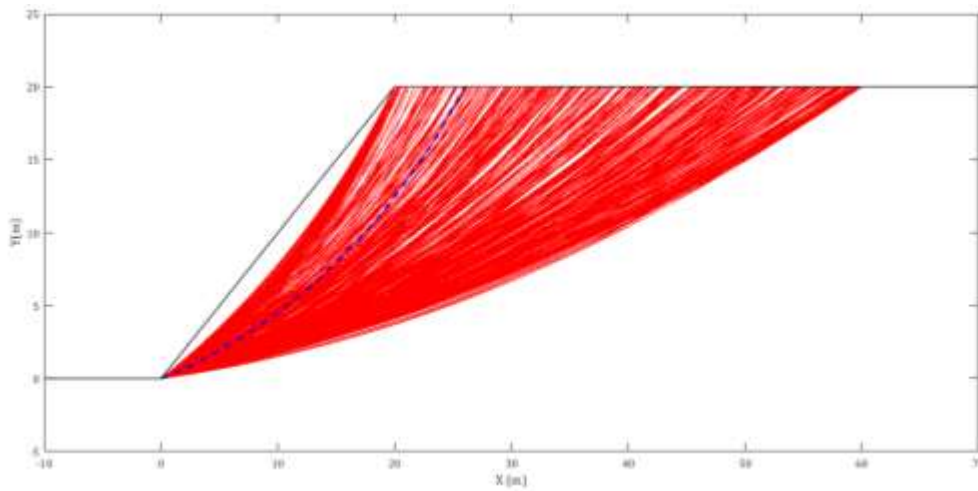


Fig.5. Using the PSO algorithm for Determining the critical failure surface of the slope

Due to the characteristics of soil slope parameters and its dimensions, suitable geogrids for soil reinforcement were selected and simulated in different layers. Figure 5 presents the geometrical characteristics of the slope reinforced with geogrid. At a height of 20 m, the geogrids are fixed at 3 m to 3 m from the top of slope and the distance between them is assumed to be the same. As we know with decreases the slope height, the number of layers needed for reinforcement decreases.

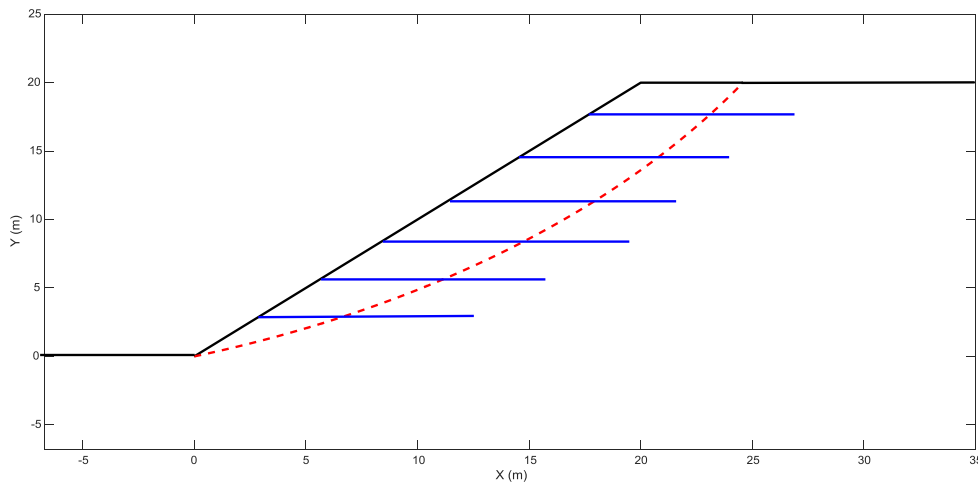
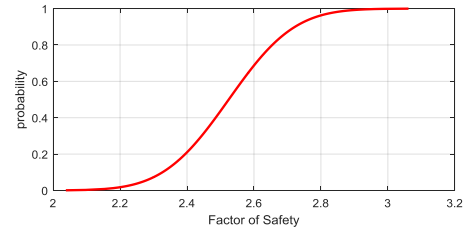
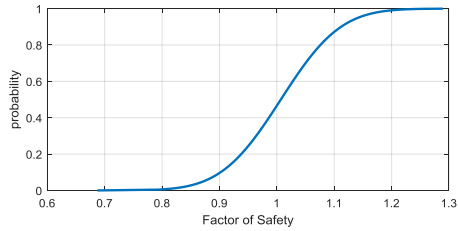
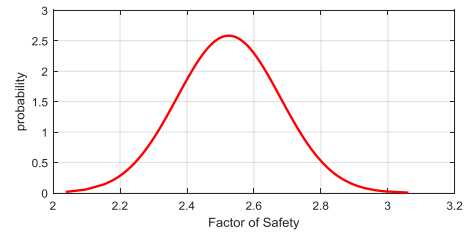
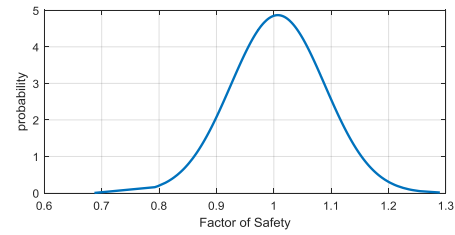


Fig.6. Schematic of a reinforcement slope with a height of 2 meters



a) With out geogrid

b) with geogrid

Fig.7. Probability density function and Cumulative distribution Function of safety factor for MCS stability analysis of height 20 m

The final probability density functions for the safety factor are determined using MCs. For this purpose, 10,000 generations are used for the MCs. The results are shown in Fig. 7 for ordinary method of slices. Fig. 7.a shows the Probability Density Function (PDF) and Cumulative distribution function (CDF) of safety factor based on total stresses for a slope with out reinforcement and Fig. 7.b shows this characters for reinforcement slope. The reliability index of slope according to factor of safety and MCS is determined using the PDF and the following equation [41].

$$\beta = \frac{E(FS) - 1}{\sigma(FS)} \quad (19)$$

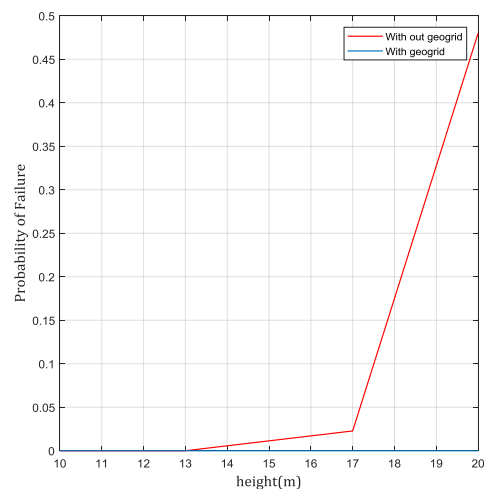
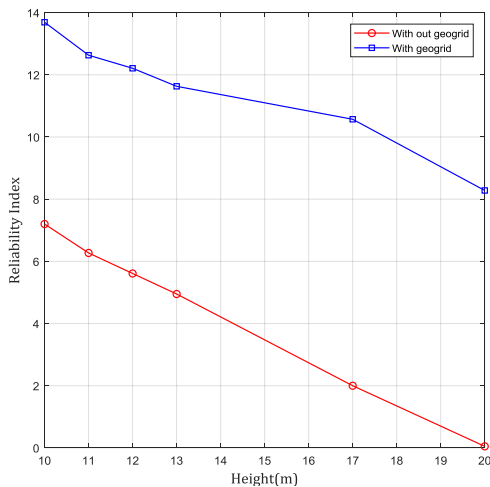


Fig.8. Contribution of probability of failure with respect to the slope height



Table 2. The result of calculation of height 20 m

	FS	Reliability index(β)
Without geogrid	1.00	0.00
with geogrid	2.58	9.28

Fig. 8 presents the variations of the reliability index and probability of failure of the slope with respect to its height. For this purpose, a slope altitude range of 10-20 m is considered and reliability analysis for each height is performed in two case, reinforced and normal slope states. As can be seen in Fig. 8, the reliability index is in the safe range on the reinforced slope, but the slope without reinforcement at height of 20 m is near to be failed and the probability of failure is greatly increased. Finds while the reinforced slope is quite safe and almost at the probability failure of 0. Table 3 indicates that based on determined reliability index the reinforced slope has the high expected performance level.

Table 3. Target reliability indices [23]

Expected performance level	Reliability index(β)	Failure Probability of (Pf)
High	5.0	$0.30 \times 10^{-6.0}$
Good	4.0	$0.30 \times 10^{-4.0}$
Above average	3.0	$0.10 \times 10^{-2.0}$
Below average	2.5	$0.60 \times 10^{-2.0}$
Poor	2.0	$0.23 \times 10^{-1.0}$
Unsatisfactory	1.5	0.07
Hazardous	1.0	0.16

CONCLUSION 6.

Geosynthetics as novel materials used in engineering have great potential that can be used as cost-effective solutions to several engineering problems. This paper presents one of recent advances in geosynthetic products, their use in soil reinforcement, and in environmental applications. The production of geosynthetic allows for newer and better solutions to recent advances in engineering. Therefore, it is expected that innovation in products, types and properties will continue and add to the widespread applications of these materials.

This paper presents a numerical comparison of the slope reliability index in reinforced and unreinforced states. To do this, a code was written in the Matlab software that calculates the reliability index using the Monte Carlo simulation. As observed, the slope stability analysis was performed at a variable height of 10–20 m, and it was observed that the slope reached the fracture boundary with increasing height and its reliability index tended to zero. However, the reinforced slope retains its stability with increasing height, and its reliability index remains at a high expected performance level up to 45 °. Also the results of Monte Carlo simulation indicate that the mean values of the probability density function are consistent with the results of the deterministic analysis.

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